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Rheological factors in a thermal problem of local superhigh frequency hyperthermia

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Abstract—The effect of a rheological factor on a temperature field in a heterogeneous tissue in its SHF heating is considered. A microvascular system is presented in the form of quasiporous media and is replaced by an equivalent tube. The relation is found between relative change in perfusion, the parameter of blood viscoplasticity and hydraulic diameter of an equivalent tube. Perfusion is found to be proportional to the cube of a hydraulic diameter of this tube and conversely proportional to the value of the parameter of viscoplasticity. Their possible values and the range of variation with local hyperthermia (LH) are estimated. It is shown that the values of the parameter of viscoplasticity within the range 1–0.06 correspond to small shear rate (5–100 s^{-1}). The changes in the hydraulic diameter in a normal tissue in LH may attain 20–80%, the changes in the parameter of viscoplasticity may vary by 2.5 and more times. It is shown on the example of frequencies 2450 and 915 MHz that the rheological factor of blood flow exerts a very substantial effect on a temperature field in tissue, especially in the presence of a slightly perfused tumour in it.

1. INTRODUCTION

One of the trends of the investigation of heat and mass transfer processes in biological objects is the study and modeling of hemodynamics with hyperthermia of malignant tumours. Urgency in these studies is stipulated by the necessity of sustaining rather a narrow range of temperatures in tissue with LH and by a large value of a convective component of heat sinks in a heated region.

As is known [1], the most effective modes of tumour hyperthermia are attained at temperatures 42–45°C (the therapeutic range). These conditions may be realized by varying the power of super high frequency (SHF) radiation and heat transfer on the outer surface. Thermal processes in tissue are then determined by electromagnetic and thermophysical characteristics (thermal conductivity, heat capacity, density), heat carried away from tissue by a blood flow (perfusion) and heat convection into a surrounding medium.

A determining role in this case is played by perfusion, i.e. by blood flow rate in a time unit per 100 g of tissue. As is known [2], with local hyperthermia in different tissues being heated, perfusion may increase by an order of one or more. A strict description of this temperature dependence of a blood flow requires a correct allowance for a rheological factor and hemodynamic characteristics of a blood–vascular system. This is the aim of the present paper.

2. MODELING OF A RHEOLOGICAL FACTOR

A great number of papers are devoted to modeling a blood flow. They, as a rule, consider a blood flow in a separate cylindrical channel with rigid [3, 4] or elastic [5] walls, impermeable or permeable [6–8], under the conditions of constant or fluctuating pressure [9]. This description does not solve the problem of the evaluation of a blood flow in an animate tissue or in an organ due to the extremely complex and diverse architectonics of a microvascular system, uncertainty and variability of hemodynamic parameters, in particular, geometric and rheological.

An approach based on the model of a quasiporous medium, through which a non-Newtonian fluid is filtered, seems to be promising.

For non-round channels and porous impermeable media use is made of the concept of a round equivalent tube providing, with the same head, the flow rate equal to that in the channel or a porous medium. A hydraulic diameter is determined by the relation $D = 4S/\Pi$, where S is the cross-sectional area of a non-round channel or a porous medium, Π is the wetted perimeter [10].

The relation between a hydraulic diameter and the geometry of a flow through system and other characteristics, e.g. blood filling, is a complex independent problem. In the present paper we only use a concept of an equivalent tube for a simplified description of the relative change in perfusion with LH.

Consider the effect of the Iliyushin plasticity parameter on tissue heating for the Shvedov–Bingham state

$$Il = \frac{\tau_0 \cdot D}{\mu_p \bar{u}}. \quad (1)$$

Blood as a suspension of forming elements (leukocytes, erythrocytes, etc.) in plasma is strongly non-Newtonian. A rheological behaviour of a non-linear

NOMENCLATURE

a	Bouger coefficient of absorption of electromagnetic energy [m^{-1}]
Bi	Biot number, $\alpha \cdot H/\lambda$
C	heat capacity [J kg · K^{-1}]
D	hydraulic diameter [m]
f	perfusion [ml · (100 g tissue min)$^{-1}$]
h	current coordinate (depth) [m]
H	depth of a heated layer [m]
Il	Iliyushin parameter, $\tau_0 D/(\mu_p \bar{u})$ [non-dimensional]
L	length of effective tube [m]
n, m	dimensionless nonlinearity parameters of a generalized rheological model of viscoplastic medium
P	power of a SHF-emitter [W]
Q	volumetric blood flow rate through an effective tube [$m^3 s^{-1}$]
q_0	surface density of electromagnetic radiation power on skin [W m^{-2}]
R	radius of effective tube [m]
S	Saint-Venan parameter, $\tau_0 D^{n/m}(\mu_p \bar{u})^{-n/m}$ [non-dimensional]
s, d, b, c	parameters of SHF-emitter [kg^{-1}, m, m^{-1}, m, respectively]
T	temperature [°C]
T_0	homeostasis temperature [°C]
T_s	temperature of cooling fluid [°C]
$\bar{u}$	mean linear velocity of flow in a tube [m s^{-1}]
W_b	blood mass flow rate [kg $m^{-3} s^{-1}$]
x	dimensionless coordinate layer depth, h/H.

Greek symbols

Θ	dimensionless temperature, $(T-T_0)/(T_0-T_s)$
α	coefficient of heat transfer on skin surface [W $m^{-2} K^{-1}$]
$\dot{\gamma}$	shear rate [s^{-1}]
$\langle\dot{\gamma}\rangle$	shear rate averaged over tube cross section [s^{-1}]
ζ	relative thickness of a quasisolid core of a viscoplastic flow, τ_0/τ_w
λ	thermal conductivity [W $m^{-1} K^{-1}$]
μ_p	analog of plastic viscosity [N m^{-2} s]
ρ	density [kg m^{-3}]
τ_0	yield stress [N m^{-2}]
τ_w	shear stress at a tube wall [N m^{-2}]
τ	shear stress [N m^{-2}].

Subscripts

b	refers to blood
in	initial value of a parameter.

viscoplastic medium in a general case [11] is described by the governing equation

$$\tau^{1/n} = \tau_0^{1/n} + (\mu_p \dot{\gamma})^{1/m}. \quad (2)$$

Now, in biomedical and clinical studies the Casson model, a particular case of equation (2) at $n = m = 2$, is the most widespread one.

The curve of the Casson flow is approximately valid within the range of shear rates 0.01–20 s^{-1} [22] for normal blood in the absence of thermal, chemical, radiate and other effects. Deviations from the Casson curve are observed beyond this range and in pathological cases (ischemic disease, diabetes, diseases of blood itself, etc.). Biorheological studies of the last decade, conducted in Moscow, Kiev, Minsk and Nizhniy–Novgorod, showed non-adequacy of the Casson flow curve (CFC) to the real mechanical behaviour of normal blood. Figure 1 depicts Sundukov's processing of his own rheometric experiments [13]. Three pairs of the values of non-linear viscoplasticity were used in a general law (2): (a) $n = m = 1$, the Shvedov–Bingham fluid (a linear model); (b) $n = m = 2$, the Casson equation; and (c) $n = m = 3$, a more general rheological dependence. Based on these data a real curve of a normal blood flow is better approximated by equation (2) with $m = n = 3$ than by the Casson model.

Any pathology, in principle, affects the rheological behaviour of blood, this is indirectly expressed by the change in the rate of erythrocyte precipitation. Consequently, in many diseases the curve of blood flow may be characterized by $n \neq m$, within the ranges $1 \leqslant n \leqslant 3$, $1 \leqslant m \leqslant 3$; in this case the rheological par-

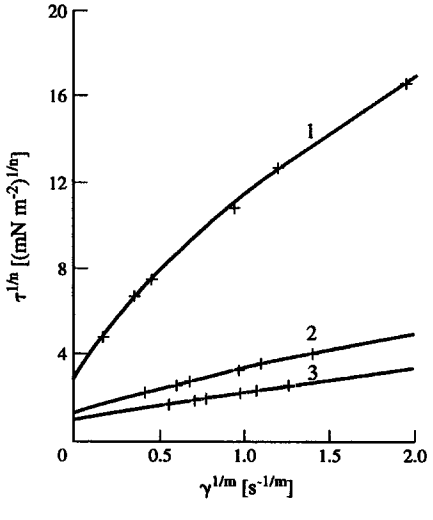

Fig. 1. Blood rheometry by A. N. Sundukov. 1, $n = m = 1$; 2, $n = m = 2$; 3, $n = m = 3$.

ameters n and m may not be integers (with the rule of signs of τ and $\dot{\gamma}$ being observed).

With a stabilized pressure flow of any inelastic non-Newtonian fluid in a cylindrical tube the flow rate Q and the flow curve $f(\tau)$ are related by the Mooney–Rabinovich universal dependence [11]

$$\frac{Q}{\pi R^3} = \frac{1}{\tau_w^3}\int_{\tau_0}^{\tau_w} \tau^2 f(\tau)\,d\tau \tag{3}$$

where R is the tube radius, τ_w is the shear stress on the tube wall.

In the considered cases

$$f(\tau) = (\tau^{1/n} - \tau_0^{1/n})^m \mu_p \tag{4}$$

where $n = m = 3$ or $n = m = 2$. After the substitution of equation (4) into (3) and subsequent integration obtain

$$Q_{n,m}/Q_N = \varphi_{n,m}(\zeta) \tag{5}$$

where

$$\varphi_{n=m=2}(\zeta) = 1 - \tfrac{16}{7}\zeta^{1/2} + \tfrac{4}{3}\zeta - \tfrac{1}{21}\zeta^4, \tag{6}$$

$$\varphi_{n=m=3}(\zeta) = 1 - \tfrac{36}{11}\zeta^{1/3} + \tfrac{36}{10}\zeta^{2/3} - \tfrac{4}{3}\zeta + \tfrac{1}{165}\zeta^4. \tag{7}$$

Here $\zeta = \tau_0/\tau_w = r_0/R$ is the relative thickness of a quasisolid core, $Q_N = \pi R^4 \Delta P/8\mu_p L$ is the Poiseuille flow rate of a Newtonian fluid with the viscosity μ_p through a cross-section of a cylindrical tube.

Equation (5) corresponds the flow rate $Q_{n,m}$ to the Newtonian analog. In a general case relation (5) does not match a relative change of blood flow (perfusion f or mass blood flow rate W_b) before and after the run of hyperthermia because blood is of a non-Newtonian character even before heating.

Rheological description of blood flow by the Iliyushin parameter is incomplete. A non-Newtonian character of blood manifests itself also in the absence of structure formation, i.e. at $\tau_0 = 0$. In this case viscosity remains reduced even with the growth of the shear rate and blood behaves as a non-linear viscous fluid.

Simultaneous and more complete account for non-linear viscosity and viscoplasticity is achieved at n and m being not equal to each other. However, the quantity τ_0, as a structural characteristic of blood, has a determining value both in normal and pathological states. That is the reason for the analysis based on the Iliyushin parameter to be of interest.

In a general case of $n \neq m$ there takes place the complex including the parameters of viscous, m, and plastic, n, non-linearities

$$S = \frac{\tau_0 \cdot D^{n/m}}{\mu_p^{n/m} \cdot \bar{u}^{n/m}} \tag{8}$$

that changes over to the parameter Il at $n = m$.

The Iliyushin criterion (1) and formula (3) yield the expression for the relative change in the fluid flow rate through a cylindrical channel at arbitrary positive n and m.

If n and m are the integers, then

$$f(\tau) = \frac{1}{\mu_p}(\tau^{1/n} - \tau_0^{1/n})^m = \frac{1}{\mu_p}\sum_{k=0}^{m}(-1)^k C_m^k \tau^{(m-k)/n}\tau_0^{k/n}.$$

Having substituted this expression into (3), we obtained (see [11])

$$Q = \frac{n}{3n+m}\frac{\pi R^3}{\mu_p}\tau_w^{m/n}\varphi_{n,m}(\zeta) \tag{9}$$

where

$$\varphi_{n,m}(\zeta) = \sum_{k=0}^{m}(B_k/B_0)(\zeta^{k/n} - \zeta^{m/n+3}) \tag{10}$$

$$B_k = \frac{(-1)^k C_m^k}{3+(m-k)/n}.$$

If n and m are rational numbers, then, having expanded $f(\tau)$ into a series of τ and substituted it into (3), we obtain the expression for the flow rate similar to equation (9) (see [11]), where

$$\varphi_{m,n}(\zeta) = \sum_{k=0}^{\infty}(B_k/B_0)(\zeta^{k/n} - \zeta^{m/n+3}).$$

Note, that

$$\varphi_{n,m}(\tau_0 = 0) = 1; \quad \varphi_{n,m}(\tau_w = \tau_0) = 0.$$

From the determination of criterion (8) and formula (3) have

$$S = \frac{\tau_0(2R)^{n/m}}{\mu_p^{n/m}(Q/\pi R^2)^{n/m}} = (2(3+m/n))^{n/m}\frac{\zeta}{\varphi_{n,m}^{n/m}(\zeta)}. \tag{11}$$

For relative variation of flow rate obtain the expression

$$\frac{Q}{Q_{in}} = \left(\frac{R}{R_{in}}\right)^3 \frac{\mu_{p,in}}{\mu_p}\left(\frac{\tau_w/\tau_0}{\tau_{w,in}/\tau_{0,in}}\right)^{m/n}\left(\frac{\tau_0}{\tau_{0,in}}\right)^{m/n}\frac{\varphi_{n,m}(\zeta)}{\varphi_{n,m}(\zeta_{in})}$$

$$= \left(\frac{R}{R_{in}}\right)^3 \frac{\mu_{p,in}}{\mu_p}\left(\frac{\zeta_{in}}{\zeta}\right)^{m/n}\left(\frac{\tau_0}{\tau_{0,in}}\right)^{m/n}\left(\frac{\zeta}{\zeta_{in}}\right)^{m/n}\left(\frac{S_{in}}{S}\right)^{m/n}$$

and, thus

$$Q/Q_{in} = (R/R_{in})^3(\mu_{p,in}/\mu_p)(\tau_0 S_{in}/\tau_{0,in}S)^{m/n}. \tag{12}$$

In deriving equation (12) it was assumed that the coefficients n and m are constant.

In a particular case of $n = m$ the criterion S is converted to the Iliyushin criterion Il and expression (10) is transformed to equation (6) at $n = m = 2$ and to equation (7) at $n = m = 3$. A relative variation of the fluid flow rate is then described by the formula

$$\frac{Q}{Q_{in}} = \left(\frac{R}{R_{in}}\right)^3 \frac{\mu_{p,in}}{\mu_p}\frac{\tau_0}{\tau_{0,in}}\frac{Il_{in}}{Il}. \tag{13}$$

If $\tau_0 = 0$, then from equation (9) we have

$$Q/Q_{in} = (R/R_{in})^{3+m/n}(\mu_{p,in}/\mu_p)$$

(the latter expression is given for the case of $\Delta p/L = \text{const.}$).

We have not as yet at our disposal the data *in vivo* on the change of μ_p and τ_0 during hyperthermia. The data of rheometric studies *in vitro* of the temperature effect on blood rheological characteristics [15] indicate the reduction of μ_p and τ_0 with an increase of temperature from normal to 45°C. On the other hand, literature [14] gives the facts of an increase of fibrinogen and albumin concentrations in a tumour with heating (especially at temperatures higher than 45°C), this fact should be accompanied by the growth of μ_p and τ_0. The increase in the blood viscosity is also facilitated by the decrease of pH in a tumour [16, 17]. Taking the above into account, we assume in the first approximation

$$(\tau_0/\tau_{0,in})/(\mu_p/\mu_{p,in}) \approx 1. \qquad (13')$$

Then, a relative variation of the flow rate and thus of the mass blood flow rate may be presented as

$$\frac{W_b}{W_{b,in}} = \left(\frac{R}{R_{in}}\right)^3 \frac{Il_{in}}{Il}. \qquad (14)$$

The Iliyushin parameter, generally speaking, depends on the tube radius. However, there are experimental and theoretical data confirming the fact that in the system of blood microcirculation with normothermia (and, apparently, in the absence of muscular activity) the flow rate through a vascular tube is proportional to a cube of the diameter of this vascular tube [18, 19]. Hence follows the independence of Il on R under these conditions. Change in blood viscosity on its heating during LH may violate this conclusion (in particular, due to the fact that this viscosity itself depends on the value of an effective diameter of the vascular tube because of the Fahraeus–Lindqvist effect). Moreover, when we have to do with thermoregulation in LH, a postulate on the correspondence of vascular reactions to the minimum of energy losses for blood passing through a vascular tube used to substantiate a cube dependence of a flow rate on a vascular tube diameter seems to be questionable [19].

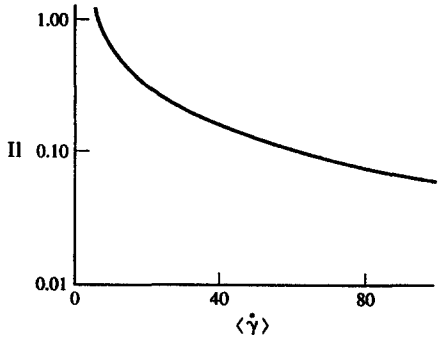

Fig. 2. Iliyushin parameter for average shear rate.

Figure 2 presents our estimate of an absolute value of the parameter Il made proceeding from the relation $Il = (\pi\tau_0/\mu_p)(1/\langle\dot{\gamma}\rangle)$ and from mean shear velocities $\langle\dot{\gamma}\rangle = 5\text{–}100\ s^{-1}$, typical of some portions of the system of blood microcirculation [20, 21]. It was taken here that $\tau_0 = 0.01\ N\ m^{-2}$, $\mu_p = 5 \times 10^{-3}\ N\ m^{-2}\ s$. Because limiting values for τ_0 do not exceed 0.05 N m^{-2} [22], then within the above range for $\langle\dot{\gamma}\rangle$ maximum values of the Iliyushin parameter may attain (in some pathologies) several tens.

According to (14) perfusion changes in proportion to the cube of a vascular tube diameter. Consequently, the ratio R/R_{in}, along with the Iliyushin criterion, determines the intensity of heat removal by blood from the heating zone with local hyperthermia.

Literature gives different data on the change of a diameter of vascular tubes under the effect of various factors, a thermal one being included.

Table 1 gives our estimates of relative changes in diameters of vascular tubes and the parameter of viscoplasticity made on the basis of equation (13) in the approximation of (13'). In those cases when use was made of the data on a vascular volume it was assumed that this volume is proportional to the square of a hydraulic diameter of an equivalent tube.

As is seen from Table 1, with LH the most considerable increases in the effective diameter of vascu-

Table 1. Vascular and hemodynamics changes with local hyperthermia

Tissue	Change of vascular volume	Change of blood flow	D/D_{in}	Il_{in}/Il	Note
Skin	2.93	—	1.71	—	rats, 43°C, 1 h [28]
	3.0	4.0	1.73	0.77	rats, 43°C, 1 h [29]
	2.42	3.73	1.56	0.99	rats, 43°C, 1 h [30]
	3.5	12.0	1.88	1.82	human, 43°C, 1 h [31]
Muscle	1.59	—	1.26	—	rats, 43°C, 1 h [28]
	1.5	3.5	1.23	1.91	rats, 43°C, 1 h [29]
	1.48	2.94	1.22	1.64	rats, 43°C, 1 h [30]
Tumour	1.06	—	1.03	—	rats, 43°C, 1 h Carcinoma Walker 256 [28]
0.3–0.7 g	1.06	1.16	1.03	1.06	rats, 43°C, 1 h [30]
2–5 g	0.97	0.90	0.99	0.91	rats, 43°C, 1 h [30]
	0.93	0.13	0.95	0.15	rats, 42.5°C, 1 h, Melanoma, A-Mel-3; arterioles [32]
	0.98	0.24	0.99	0.25	rats, 42.5°C, 1 h, Melanoma, A-Mel-3; venules [32]

lar tubes are observed in skin (by 1.6–1.9 times at temperature 43°C maintained for 1 h). Vascular of muscles under the same conditions thickens by 22–26% and the effective diameter of tumour vascular changes insufficiently both towards its increase and decrease. The latter is observed in rather large malignant new formations. These data are in accordance with the familiar postulate on considerable differences in the reaction to heating of vascular of tumour and normal tissues [23].

However, based on these data we cannot made unambiguous conclusions relative to Il_{in}/Il. One may confirm that with LH in normal tissue a decrease in Il is possible by more than two-fold, in tumor Walker 256 the parameter Il changes slightly and in melanoma A-Mel-3 a 4–6-fold growth of this parameter is possible.

3. HEAT PROBLEM

Let us approximately estimate the effect of the rheological factor on the temperature field in tissue on attaining a stationary state. To simplify the analysis proceed from a one-dimensional (1D) biothermal equation [1]. Let an electromagnetic field heat a layer of a uniform tissue. Its outer surface (skin) is cooled by fluid having a temperature T_s and a coefficient of heat transfer α, and at a depth H, $T' = 0$ is prescribed (Fig. 3).

$$\left.\begin{aligned} &\lambda\frac{d^2T}{dh^2}+q_R-q_b+q_m=0\\ &-\lambda\frac{dT}{dh}\bigg|_{h=0}=\alpha(T_s-T|_{h=0})\\ &\frac{dT}{dh}\bigg|_{h=H}=0 \end{aligned}\right\}. \tag{15}$$

The density of internal heat sources produced in tissue by SHF radiation is described by the expression for a plane electromagnetic wave [24]

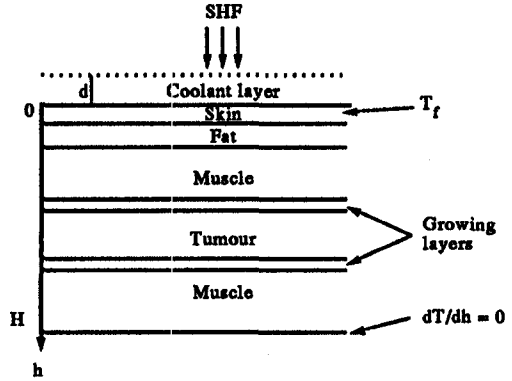

Fig. 3. Scheme for solving a biothermal problem.

$$q_R=\rho\cdot s\cdot P\cdot\exp(a(h-d))\exp(by^2/(1+c)). \tag{16}$$

This relation contains the parameters (a, b, c, d, s), dependent on the geometry and frequency of an emitter. Since we estimate the effect of a rheological factor on the example of a 1D problem ($y = 0$) and at several frequencies (2450, 915 and 463 MHz) and equal surface densities of electromagnetic radiation on a skin q_0, present q_R in the form

$$q_R=-q_0a\exp(ah). \tag{16'}$$

Determine the density of hemodynamic heat sinks by a convective condition

$$q_b=W_bC_b(T-T_0) \tag{17}$$

where C_b is the blood heat capacity, $W_b=f\rho_b\rho$ is the mass blood flow rate.

The power of metabolic heat releases in tissue during intense SHF hyperthermia may be neglected [25]. The dimensionalization of (15) leads to the system

$$\left.\begin{aligned} &\theta''-B^2\theta+R\exp(a\cdot H\cdot x)=0\\ &\theta'|_{x=0}=Bi(1+\theta|_{x=0})\\ &\theta'|_{x=1}=0 \end{aligned}\right\} \tag{18}$$

$$x=\frac{h}{H}\quad Bi=\frac{\alpha H}{\lambda}\quad\theta=\frac{T-T_0}{T_0-T_s}\quad B^2=\frac{W_bC_bH^2}{\lambda}$$

where

$$R=\frac{-q_0aH^2}{\lambda(T_0-T_s)}. \tag{19}$$

A common solution of the problem has the form

(1) when $B\neq|aH|$:

$$\theta=C\exp(-Bx)+D\exp(Bx)+E\exp(aHx) \tag{20}$$

where

$$E=\frac{R}{B^2-(aH)^2};$$

$$C=-\frac{[Bi(1+E)-EaH]B\exp(B)+EaH(B-Bi)\exp(aH)}{(B+Bi)\exp(B)-(B-Bi)\exp(-B)}$$

$$D=-\frac{[Bi(1+E)-EaH]B\exp(-B)-EaH(B+Bi)\exp(aH)}{B[(B+Bi)\exp(B)-(B-Bi)\exp(-B)]}$$

(2) at $B=|aH|$:

$$\theta=C\exp(-Bx)+D\exp(Bx)+E(1+2x)\exp(-Bx) \tag{21}$$

where

$$E = \frac{R}{4B^2};$$

$$C = -\frac{Bi(1+E) - BE + E(B-Bi)(1-2B)}{(B+Bi)\exp(B) - (B-Bi)\exp(-B)}\exp(-B)$$

$$D = -\frac{Bi(1+2E) - 2BE(B+Bi)}{(B+Bi)\exp(B) - (B-Bi)\exp(-B)}\exp(-B).$$

Figure 4 presents the dependencies of a maximum temperature of a tissue on a rheological factor at the frequencies of an emitter 2450, 915 MHz and $R = 1.2R_{in}$. The value $W_{bin} = 1$ kg m^{-3} s^{-1} (~ 5.4 ml 100 g^{-1} min^{-1}) is taken as an initial one. Computations were performed with a surface density of energy flux corresponding to the threshold of skin pain sensation equal to 0.188 W cm^{-2} [26], $\lambda = 0.4$ W m^{-1} K^{-1}, $\alpha = 20$ W m^{-2} K^{-1}, $C_b = 3640$ J kg^{-1} K^{-1}; $\rho_b = \rho = 1050$ kg m^{-3}. Coefficient a was determined for the considered frequencies by the data of [27]. In a muscle it amounted to -118 m^{-1}, -66 m^{-1} and -56 m^{-1} on the frequencies 2450, 915 and 463 MHz, respectively. These values were taken as well for skin and tumour. In fatty tissues for the same frequencies Bouger coefficients of electromagnetic radiation absorption are -17.8, -11.3 and -7.6 m^{-1}.

It is seen from Fig. 4 how considerable is the effect of a rheological factor (change in temperature with a four-fold decrease of the viscoplasticity parameter reaches 5–7 K). The decrease of frequency with the same change in the Ilyushin parameter and a fixed surface density of an electromagnetic radiation flux results in the reduction of maximum temperature in a homogeneous tissue.

The change of the depth at which maximum warming-up is reached is shown in Fig. 5. A general tendency, independently of emitter radiation, is a weak dependence of this depth on the parameter of viscoplasticity. The reduction of the frequency of SHF-radiation shifts temperature maximum into a depth

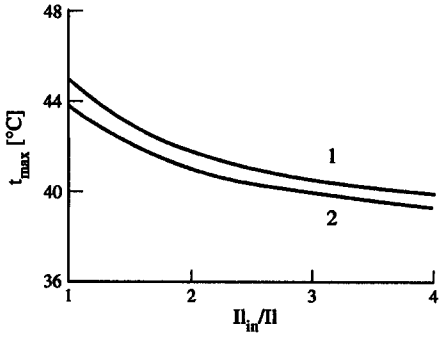

Fig. 4. Maximum temperature of tissue with the value of relative changes of viscoplasticity parameter. 1, frequency 2450 MHz; 2, 915 MHz.

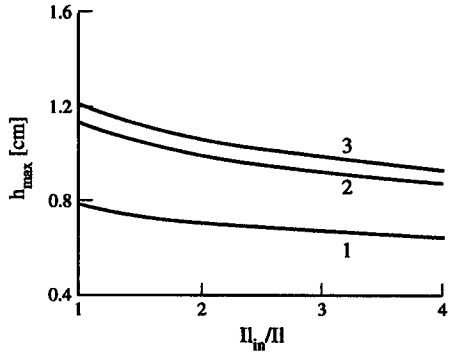

Fig. 5. The depth of maximum heating of tissue with the value of relative changes of viscoplasticity parameter. 1, frequency is 2450 MHz; 2, 915 MHz; 3, 433 MHz.

of tissue. According to calculations, this is also facilitated by the increase in the intensity of skin cooling.

Thus, the results of numerical simulation of perfusion as well as of warming-up of a homogeneous tissue indicate a drastic influence of the rheological factor of a blood flow on the hyperthermia process.

The above analysis for a homogeneous tissue is not exhaustive. Real tissue is substantially anatomically inhomogeneous as well as due to the presence of a tumour in it. It is interesting to analyze the problem in this case.

Consider a plane multilayer tissue with a 1D stationary temperature field. In this case the problem formulation is similar to equations (15)–(17), however, to the conditions on the tissue outer and inner boundaries, the conjugation conditions (equality of temperatures and heat fluxes) at the boundaries of layers are added. Thus, in the dimensionless form the problem is:

$$\left.\begin{aligned} &\theta_k'' - B_k^2 \cdot \theta_k + R_k \exp(a_k H x) = 0 \\ &\theta_k'|_{x=x_k} = (\lambda_{k+1}/\lambda_k)\theta_{k+1}'|_{x=x_k} \quad k = 0, 1, \ldots n-2 \\ &\theta_k|_{x=x_k} = \theta_{k+1}|_{x=x_k} \quad k = 0, 1, \ldots n-2 \\ &\theta_{n-1}'|_{x=1} = 0 \end{aligned}\right\}. \tag{22}$$

Here k is the number of a layer (the outer layer has a number 0, the number of the inner layer is $n-1$);

$$B_k^2 = \frac{W_b C_b H^2}{\lambda_k} \qquad Bi = \frac{\alpha H}{\lambda_0}.$$

A general solution of this problem has the form

$$\theta_k(x) = C_k \exp(-B_k x) + D_k \exp(B_k x) + E_k \exp(a_k H x) \tag{23}$$

where

$$E_k = \frac{R_k}{B_k^2 - (a_k H)^2} \quad (B_k \neq |a_k H|)$$

and the coefficients C_k and D_k are found from the boundary and conjugate conditions by solving the

obtained system of $2n$ linear algebraic equations (SLAE):

$$S \cdot G = R \tag{24}$$

where

$$g_{2j} = C_j \quad g_{2j+1} = D_j \quad j = 0, 1, \ldots n-1$$

$$S_{00} = -(B_0 + Bi) \quad S_{01} = B_0 - Bi$$

$$S_{0k} = 0 \quad 1 < k < 2n-1$$

$$S_{2n-1,k} = 0, \quad k < 2n-2 \quad S_{2n-1,2n-2} = \exp(-B_{n-1})$$

$$S_{2n-1,2n-1} = \exp(B_{n-1})$$

$$S_{i,j} = -B_k \exp(-B_k x_k) \quad S_{i,j+1} = B_k \exp(B_k x_k)$$

$$S_{i,j+2} = B_{k+1}(\lambda_{k+1}/\lambda_k) \exp(-B_{k+1} x_k)$$

$$S_{i,j+3} = B_{k+1}(\lambda_{k+1}/\lambda_k) \exp(B_{k+1} x_k) \tag{25}$$

$$j = [i/2] \quad i = 2k+1 \quad 0 \leqslant k < n-1.$$

The rest of $S_{i,j} = 0$

$$r_0 = Bi(1 + E_0) - a_0 H E_0$$

$$r_{2n-1} = -E_{n-1} \exp(a_{n-1} H)$$

$$r_{2k+1} = (E_{k+1}\lambda_{k+1}/\lambda_k - E_k) a_k H \exp(a_k H x_k)$$

$$r_{2k+2} = (E_{k+1} - E_k) \exp(a_k H x_k) \quad k = 0, 1, \ldots n-2. \tag{26}$$

Temperatures were found from equation (23) with the coefficients C_k and D_k obtained from the solution of SLAE (24). Thermophysical and hemodynamic characteristics of tissues were taken according to [1] and are gathered in Table 2. The case of a complete absence of a blood flow in the tumour center (a necrotized core) was also considered.

The rate of a blood flow in a stationary thermal state was estimated by equation (14). In this case relative change of the value of a rheological factor varied within the limits $Il_{in}/Il = 1$–4 in a normal tissue and $Il_{in}/Il = 1$ in a tumour and R/R_{in} was assumed equal to 1 for a tumour, 1.8 for skin and 1.2 for a muscle and a fat layer.

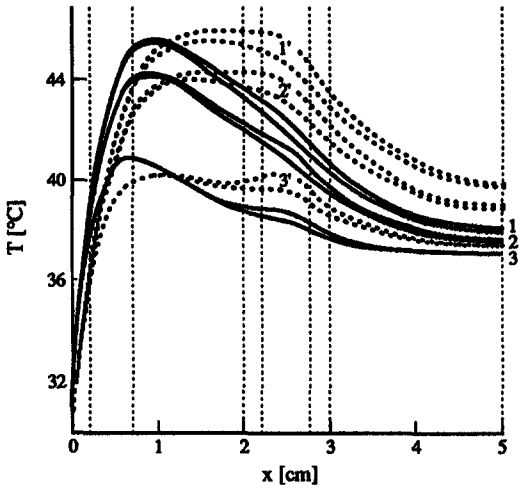

Fig. 6. Temperature distribution in a multilayer plane tissue in the presence and absence of blood flow in the tumour core (lower and upper curves bounding the dashed regions, respectively). 1, $Il = Il_{in}$; 2, $Il = Il_{in}/2$; 3, $Il = Il_{in}/3$.

Figure 6 shows the calculated temperature distributions over the depth of a tissue for the cases of necrotized and relatively weakly perfused tumour core (lower and upper curves bounding cross-hatched regions, correspondingly) at $Il_{in}/Il = 1$, 2 and 4. The frequencies 2450 and 915 MHz were considered.

As is seen from Fig. 6 the effect of the necrotized core is reduced to some temperature growth which is the most substantial in the tumour center (~ 1°C at $Il_{in}/Il = 1$; ~ 0.8°C at $Il_{in}/Il = 2$ and ~ 0.4°C at $Il_{in}/Il = 4$).

Temperature maximum at the given values of the initial parameters is attained in the center of a tumour at the frequency 915 MHz and beyond it (closer to skin) at the frequency 2450 MHz. A considerable effect of a rheological factor on temperature levels in a tissue is noticed.

4. CONCLUSIONS

(1) A maximum temperature difference with a not very large depth of tumour position and the choice of an appropriate frequency is observed in the center of a tumour.

Table 2. The parameters of the problem

Tissue	ρ [kg m^{-3}]	λ [W m^{-1} K^{-1}]	Layer depth [mm]	W_0 [10^{-3} s^{-1}]	D/D_0
Skin	1000	0.376	2	1.63	1.8
Fat	850	0.45	5	0.15	1.2
Muscle	1050	0.642	5	0.52	1.2
Tumour					
Growing layer	1050	0.502	1	1.3	1
Core	1050	0.502	10	0.52/0.13	1

(2) A four-fold increase in the Iliyushin parameter causes a considerable growth of temperature (to 4°C) in a tumor.

(3) Temperature in a necrotized core is by 0.4–1°C higher than in a perfused one.

(4) A rheological factor of a blood flow exerts a substantial effect on a maximum value and the degree of inhomogeneity of temperature distribution in a tumour and normal tissue.

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